The Lattice Boltzmann Equation for Modelling Arterial Flows: Review and Application

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Abstract

The lattice Boltzmann model is a relatively new development in computational fluid dynamics. Here we review the technique with particular emphasis on its application to biological systems. Further, we consider its application to arterial flows and discuss its potential for simulating flow on length-scales where traditional numerical approaches can be troublesome. Finally we present results from a preliminary investigation which demonstrate the suitability of the lattice Boltzmann model for simulating oscillatory flow.

Introduction

The study of arterial blood flow has many important applications. For example, the buildup of plaque deposits on blood vessel walls and its subsequent rupture can lead to sudden ischaemic cardiac death. It is therefore important to understand how blood flow affects both the buildup of the deposits and the rupture. In many cases *in vivo* measurements are extremely difficult and so numerical simulation and *in vitro* techniques become the main investigative tools.

Traditionally numerical techniques such as the finite element or finite difference methods have been applied, however to model arterial blood flow there are a number of important features which must be considered. Firstly, the blood flowing in the artery is not a simple Newtonian fluid; it transports particles such as red and white blood cells and this needs to be simulated in any realistic model. Secondly, the artery wall is far from being the solid no-slip boundary which is applied in most computational fluid dynamics (CFD) simulations. In reality the endothelium, which lines the blood vessels, contains many complex features from a computational view point. It is not smooth or regular, and it is not stationary in time due to the compliant nature of the blood vessel and the cells which form the endothelium can move over time. Further, the geometry of the artery can change significantly due to the build up of plaque, a complex process which may depend on shear stress rates in the fluid. Thus any realistic model must be able to simulate flows which have complicated evolving boundaries and which containing suspended particles. It is also desirable that the models are capable of simulating a range of length scales down to the micron sized arterioles. Here we consider a novel approach to fluid simulation: the lattice Boltzmann model (LBM) which can incorporate the different features which are required.

Features of the Lattice Boltzmann Model

The LBM [1] has recently been developed as an alternative numerical method for studying fluid flow. Despite the wide application of the LBM there have been relatively few studies of blood flow. The most noticeable exception to this is the work of Krafczyk *et al.* [2,3] who consider flow through an artificial aortic valve. Here we consider a number of features of the model which make it an attractive proposition for arterial flow simulation.

Simulation in complex geometries: An attractive feature of the LBM in any application is the simplicity with which a solid boundary can be implemented using a bounce-back boundary condition. The simplicity and effectiveness of this has been illustrated by the considerable attention which the LBM has received in the field of flow through porous media, see for example [4]. This is a clear advantage when simulating the complex geometry of the endothelium.

Compliant walls: The application of the LBM in tubes with compliant walls has recently been considered [5]. The inclusion of non-ridged walls is essential in a realistic simulation of arterial flow.

Suspended particles: Blood is not a simple fluid, it transports particles, for example red and white blood cells. LBM simulations of fluids containing suspended particles have been performed using two approaches: either directly simulating a number of particles each occupying a number of lattice grid sites [6]; or simulating the particles in a statistical sense on the same lattice as the fluid [7]. Both techniques have proved useful in simulating particle suspensions in other applications.

Micro-scale simulations: The LBM is based on a statistical representation of a fluid, rather than the continuum approach of solving the macroscopic Navier-Stokes equation. This means that it can be applied to micro-scale simulation problems. This enables simulations to be performed down to the micro-sized arterioles. The application of the LBM to micro-sized structures, and in particular to microelectromechanical systems (MEMS) has been realised by other researchers, see for example [8].

The Lattice Boltzmann Model

The LBM is based on the statistical description of a fluid in terms of the Boltzmann equation. The LBM evolves on a regular underlying grid. On this the evolution of the particle distribution functions f_i are simulated. The particle distribution function at site $\boldsymbol{r}$ and time t, $f_i(\boldsymbol{r}, t)$ is a statistical representation of the fluid particles travelling along the lattice link with label i. A typical 2D grid is shown in figure 1, where $i = 1, 2, ..., 8$ represent the links to neighbouring grid points and $i = 0$ is referred to as the rest link. The fluid density, ρ, and velocity, $\boldsymbol{u}$ are found from the particle distribution functions at each site and at each time step as

$$\rho = \sum_i f_i \quad \text{and} \quad \rho \boldsymbol{u} = \sum_i f_i \boldsymbol{e}_i .$$

The particle distribution functions evolve according to the Boltzmann equation [1]

$$f_i(\boldsymbol{r} + \boldsymbol{e}_i, t + 1) - f_i(\boldsymbol{r}, t) = \frac{-1}{\tau}(f_i - \overline{f}_i),$$

where the LHS represents streaming on the lattice and the RHS is a collision function which represents relaxation of the f_i to their equilibrium values, $\overline{f}_i$. At each site and at each time step the equilibrium distribution functions are given by [9]

$$\overline{f}_i(\boldsymbol{r}, t) = \rho \left(w_i + 3 w_i \boldsymbol{e}_i \cdot \boldsymbol{u} + \frac{9}{2} w_i (\boldsymbol{e}_i \cdot \boldsymbol{u})^2 - \frac{3}{2} w_i u^2 \right),$$

and $w_0 = 4/9$, $w_1 = w_2 = w_3 = w_4 = 1/9$ and $w_5 = w_6 = w_7 = w_8 = 1/36$. The relaxation time τ is a free variable in the simulation which can be used to tune the fluid viscosity.

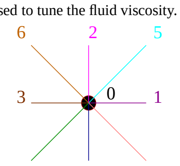

Figure 1: A 2D lattice

Simulation Results

To validate the LBM for oscillatory flow problems a number of simulations were performed in a two-dimensional channel. This problem was selected because it has a well-established analytical solution [10] which enables the LBM simulations to be vigorously verified for an oscillating flow. The results for a simulation performed for an oscillation rate of 2 Hz and a maximum central velocity of 10 cms^{-1} in a channel of width 9 mm is shown in figure 2 and compared to the analytic solution. The simulation was performed with rigid boundaries for a fluid with density 1000 kgm^{-3} and viscosity 4 mPa·s.

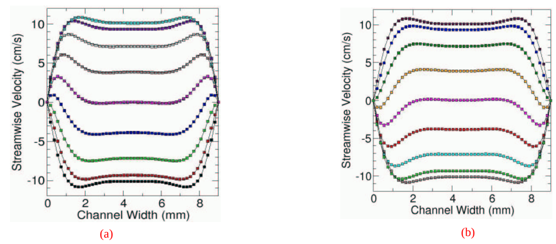

Figure 2: LBM simulations of oscillating flow (x) for (a) upstroke and (b) downstroke. Also shown is the analytic solution (line).

The LBM is also capable of directly simulating turbulence which can occur in the larger arteries. Figure 3 shows a turbulent channel flow over one period T of the oscillatory motion. The channel geometry is the same as in figure 2, however a larger flow rate is applied to achieve a turbulent regime. It was found to be necessary to include a small level of noise in the simulations to trigger the turbulence. The mean energy of the noise was less than 0.1 % of the average energy of the flow and considerably smaller than the turbulent motion observed.

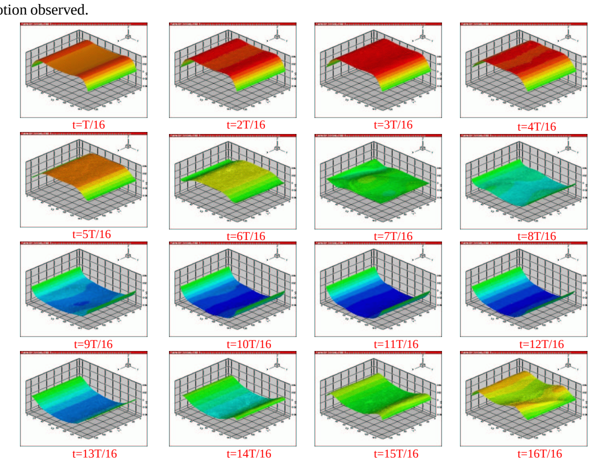

Figure 3: Simulation of turbulence.

Conclusions

We have presented a review of the LBM concentrating on the attributes which make it attractive to arterial flow simulation. The main features are highlighted below. Suspended particles can be incorporated into the flow; this is required to model particles such as red and white blood cells and also the deposition and rupture of plaque during atherosclerosis. The LBM can be applied to micro-scale flow phenomena; this is essential if a large range of physiological flows is to be studied. The ability of the LBM to accurately simulate non-stationery, oscillatory flows has also been highlighted. These features suggest that the LBM is potentially an important and useful tool for investigating a large range of arterial flow problems. The model used here is at presently being extended to three dimensions. Future work will involve adapting many of the features discussed here, as well as investigating the possibility of coupling the LBM to models incorporating plaque deposition [11].

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